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International Baccalaureate®
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FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 20 May 2010 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. *[Maximum mark: 7]*

A university Mathematics Department admits students on the basis of performance in an entrance examination which is graded 'excellent', 'very good' or 'good'. The students sit their final examination three years later when they are awarded 'first class', 'second class' or 'third class' degrees. The results for a particular group of students are summarised in the following table.

	First class	**Second class**	**Third class**
Excellent	30	14	6
Very good	8	12	5
Good	5	6	14

Stating your hypotheses, use an appropriate test to investigate at the 1 % level of significance whether or not there is an association between performance in the entrance examination and performance in the final examination. Justify your answer.

2. *[Maximum mark: 10]*

Let S be the set of matrices given by

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}; a, b, c, d \in \mathbb{R}, ad - bc = 1.$$

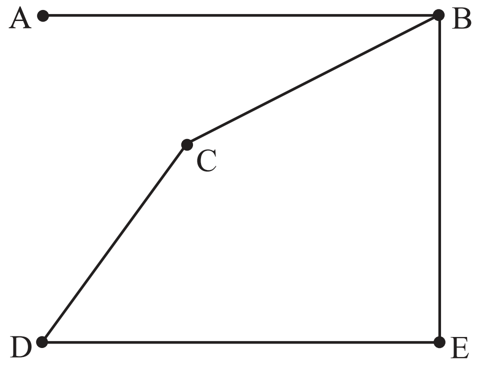
The relation R is defined on S as follows. Given $\boldsymbol{A}, \boldsymbol{B} \in S$, $\boldsymbol{A}R\boldsymbol{B}$ if and only if there exists $\boldsymbol{X} \in S$ such that $\boldsymbol{A} = \boldsymbol{B}\boldsymbol{X}$.

(a) Show that R is an equivalence relation. *[8 marks]*

(b) The relationship between a, b, c and d is changed to $ad - bc = n$. State, with a reason, whether or not there are any non-zero values of n, other than 1, for which R is an equivalence relation. *[2 marks]*

3. *[Maximum mark: 11]*

The figure below shows the graph G.



(a) (i) Write down the adjacency matrix for G.

(ii) Find the number of walks of length 4 beginning and ending at B. *[5 marks]*

(b) (i) Draw G', the complement of G.

(ii) Write down the degrees of all the vertices of G and all the vertices of G'.

(iii) Hence, or otherwise, determine whether or not G and G' are isomorphic. *[6 marks]*

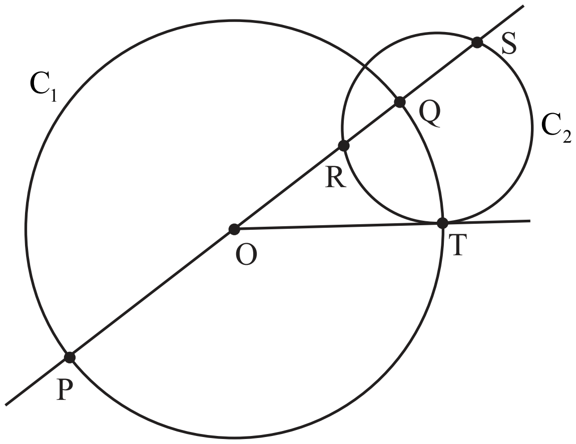
4. *[Maximum mark: 9]*

Given that $n^2 + 2n + 3 \equiv N \pmod{8}$, where $n \in \mathbb{Z}^+$ and $0 \le N \le 7$, prove that N can take one of only three possible values.

5. *[Maximum mark: 11]*

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} + 2y \tan x = \sin x$, and $y = 0$ when $x = \frac{\pi}{3}$, find the maximum value of y.

6. *[Maximum mark: 12]*



The figure shows a circle C_1 with centre O and diameter [PQ] and a circle C_2 which intersects (PQ) at the points R and S. T is one point of intersection of the two circles and (OT) is a tangent to C_2.

(a) Show that $\frac{\mathrm{OR}}{\mathrm{OT}} = \frac{\mathrm{OT}}{\mathrm{OS}}$. *[2 marks]*

(b) (i) Show that $\mathrm{PR} - \mathrm{RQ} = 2\mathrm{OR}$.

(ii) Show that $\frac{\mathrm{PR} - \mathrm{RQ}}{\mathrm{PR} + \mathrm{RQ}} = \frac{\mathrm{PS} - \mathrm{SQ}}{\mathrm{PS} + \mathrm{SQ}}$. *[6 marks]*

(c) Deduce that P, R, Q, S form a harmonic ratio. *[4 marks]*